

WRITTEN ASSIGNMENT 6

1. $y + 3x = 5$ and $2x + y = 10$

$$8 + 3(1) = 11, \quad 2(1) + 8 = 10$$

Not a solution.

2. $3x - y = 4$, $2x + 2y = 12$

$$y = 3x - 4 \quad 2x + 2(3x - 4) = 12$$

$$2x + 6x - 8 = 12$$

$$8x - 8 = 12$$

$$8x = 20$$

$$x = \frac{20}{8} = \frac{5}{2}$$

$$y = 3\left(\frac{5}{2}\right) - 4$$

$$= \frac{15}{2} - 4 = \frac{7}{2}$$

$$x = \frac{5}{2}, \quad y = \frac{7}{2}$$

3. $\begin{array}{r} 17x + y = 15 \quad | \quad 3 \\ -2x + 3y = -1 \quad | \quad 1 \end{array}$

$$21x + 3y = 45$$

$$-2x + 3y = -1$$

$$\hline 23x = 46$$

$$x = 2$$

$$7(2) + y = 15$$

$$y = 15 - 14 = 1$$

$$x = 2, \quad y = 1$$

4. $x + 2y = -4$, $3y - 2x = -13$

$$x = -2y - 4$$

$$3y - 2(-2y - 4) = -13$$

$$3y + 4y = -13 - 8$$

$$7y = -21$$

$$y = -3$$

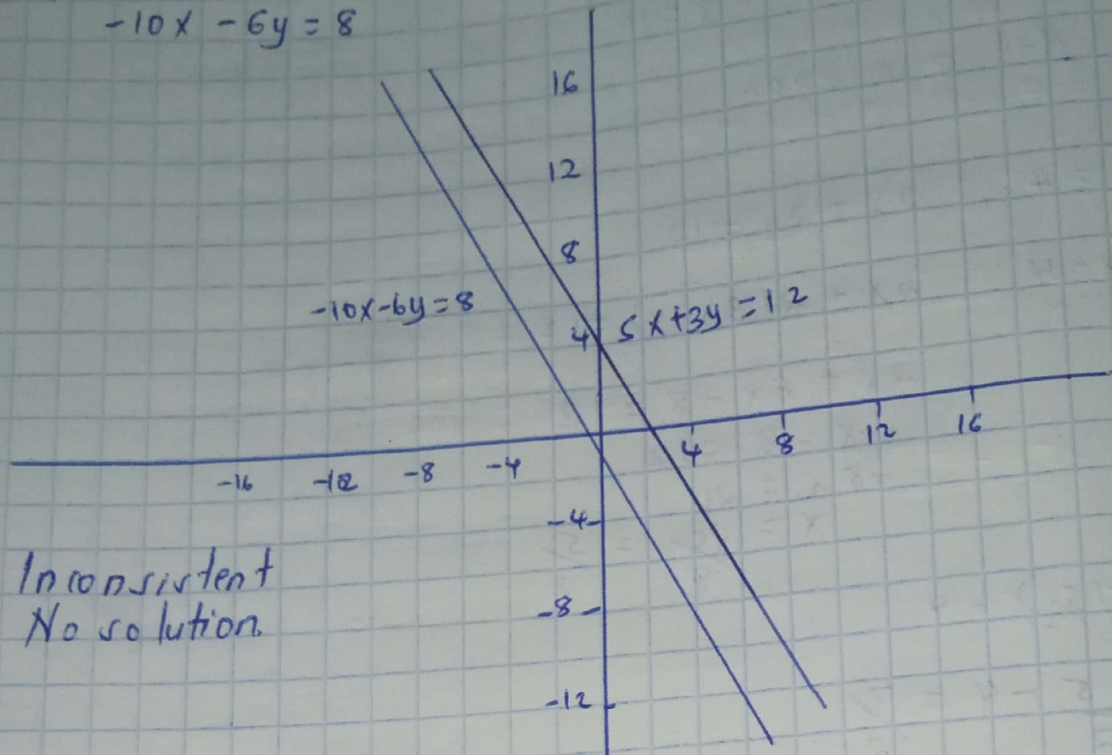
$$x = -2(-3) - 4$$

$$= 2$$

$$x = 2, \quad y = -3$$

$$5. \quad 5x + 3y = 12$$

$$-10x - 6y = 8$$



Inconsistent
No solution.

$$6. \quad C(x) = 55x + 15000$$

$$R(x) = 70x$$

$$\text{Break even: } R(x) = C(x)$$

$$70x = 55x + 15000$$

$$15x = 15000$$

$$x = 1000 \text{ tickets}$$

$$R(x) = 70 \times 1000$$

$$= 70000$$

$$7. \quad C(x) = 15000 + 17x$$

$$R(x) = 25x$$

$$25x = 15000 + 17x$$

$$8x = 15000$$

$$x = 1875$$

$$x + y = 20$$

$$0.15x + 0.48y = 20(0.35)$$

$$x + y = 20$$

$$y = 20 - x$$

$$0.15x + 0.48x = 7$$

$$0.15x + 0.48(20 - x) = 7$$

$$0.15x + 9.6 - 0.48x = 7$$

$$-0.33x = -2.6$$

$$x = 8 \text{ gallons}$$

$$y = 20 - 8 = 12 \text{ gallons}$$

$$\begin{array}{r} 14c + 2a = 133.5 \quad 2 \\ 16c + 4a = 235.5 \quad 1 \end{array}$$

$$8c + 4a = 267$$

$$6c + 4a = 235.5$$

$$2c = 31.5$$

$$c = 15.75$$

$$4(15.75) + 2a = 133.5$$

$$63 + 2a = 133.5$$

$$2a = 70.5$$

$$a = 35.25$$

Adults, \$35.25

Children, \$15.75

$$10. \quad 2(-1) - 3(1) + 4(2) = 3$$

$$-1 + 5(1) + 2(2) = 10$$

$$3(-1) - 4(1) - 2 = -9$$

It is a solution.

$$11. \quad x - 3y + 5z = 29$$

$$3x + y - 4z = -9$$

$$4x - 3y + 7z = 46$$

$$x = 29 + 3y - 5z$$

$$y + 3(29 + 3y - 5z) - 4z = -9$$

$$-3y + 4(29 + 3y - 5z) + 7z = 46$$

$$y + 87 + 9y - 15z - 4z = -9$$

$$-3y + 116 + 12y - 20z + 7z = 46$$

$$10y - 19z = -96$$

$$9y - 13z = -70$$

$$y = \frac{19z - 96}{10}$$

$$9\left(\frac{19z - 96}{10}\right) - 13z = -70$$

$$9(19z - 96) - 130z = -700$$

$$171z - 864 - 130z = -700$$

$$41z = 164$$

$$z = 4$$

$$y = \frac{19(4) - 96}{10} = -2$$

$$x = 29 + 3(-2) - 5(4) = 3$$

$$x = 3, y = -2, z = 4$$

12 $x + y + z = 6$

$$3x + 2y + 4z = 37$$

$$-4x + 3y - 6z = -53$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & -2 & 4 & 37 \\ -4 & 3 & -6 & -53 \end{array}\right)$$

$$R_2 - 3R_1 \rightarrow R_2, R_3 + 4R_1 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -5 & 1 & 19 \\ 0 & 7 & -2 & -29 \end{array}\right)$$

$$-\frac{1}{5}R_2 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -\frac{1}{5} & -\frac{19}{5} \\ 0 & 7 & -2 & -29 \end{array}\right)$$

$$R_1 - R_2 \rightarrow R_1, R_3 - 7R_2 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{6}{5} & \frac{49}{5} \\ 0 & 1 & -\frac{1}{5} & -\frac{19}{5} \\ 0 & 0 & -\frac{3}{5} & -\frac{13}{5} \end{array}\right)$$

$$-\frac{5}{3}R_3 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{6}{5} & \frac{49}{5} \\ 0 & 1 & -\frac{1}{5} & -\frac{19}{5} \\ 0 & 0 & 1 & 4 \end{array} \right)$$

$$R_3 - 1.2R_3 \rightarrow R_1, \quad R_2 + \frac{1}{5}R_3 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array} \right)$$

$$x = 5, \quad y = -3, \quad z = 4$$

$$13. \quad \frac{1}{3}x + \frac{2}{5}y + \frac{1}{9}z = 1$$

$$\frac{1}{6}x - \frac{3}{5}y + \frac{2}{3}z = 10$$

$$\frac{5}{6}x + \frac{4}{5}y - \frac{7}{9}z = -6$$

$$\left(\begin{array}{ccc|c} \frac{1}{3} & \frac{2}{5} & \frac{1}{9} & 1 \\ \frac{1}{6} & -\frac{3}{5} & \frac{2}{3} & 10 \\ \frac{5}{6} & \frac{4}{5} & -\frac{7}{9} & -6 \end{array} \right)$$

$$3R_1 \rightarrow R_1$$

$$\left(\begin{array}{ccc|c} 1 & \frac{6}{5} & \frac{1}{3} & 3 \\ \frac{1}{6} & -\frac{3}{5} & \frac{2}{3} & 10 \\ \frac{5}{6} & \frac{4}{5} & -\frac{7}{9} & -6 \end{array} \right)$$

$$R_2 - \frac{1}{6}R_1 \rightarrow R_2, \quad R_3 - \frac{5}{6}R_1 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & \frac{6}{5} & \frac{1}{3} & 3 \\ 0 & -\frac{4}{5} & \frac{11}{18} & \frac{19}{2} \\ 0 & -\frac{1}{5} & -\frac{19}{18} & -\frac{17}{2} \end{array} \right)$$

$$-\frac{5}{4}R_2 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & \frac{5}{5} & \frac{1}{3} & 3 \\ 0 & 1 & -\frac{55}{72} & -\frac{95}{8} \\ 0 & -\frac{1}{5} & -\frac{19}{18} & -\frac{17}{2} \end{array} \right)$$

$$R_1 - \frac{6}{5}R_2 \rightarrow R_1, \quad R_3 + \frac{1}{5}R_2 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{5}{4} & \frac{69}{4} \\ 0 & 1 & -\frac{55}{72} & -\frac{95}{8} \\ 0 & 0 & -\frac{29}{24} & -\frac{87}{8} \end{array} \right)$$

$$-\frac{24}{29}R_3 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{5}{4} & \frac{69}{4} \\ 0 & 1 & -\frac{55}{72} & -\frac{95}{8} \\ 0 & 0 & 1 & 9 \end{array} \right)$$

$$R_1 - \frac{5}{4}R_3 \rightarrow R_1, \quad R_2 + \frac{55}{72}R_3 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 9 \end{array} \right)$$

$$x = 6, y = -5, z = 9$$

14.

$$c + d + g = 254$$

$$g = \frac{1}{3}d$$

$$d = 40 + c$$

$$c = d - 40$$

$$d - 40 + d + \frac{1}{3}d = 254$$

$$2d + \frac{1}{3}d = 294$$

$$6d + d = 882$$

$$7d = 882$$

$$d = 126$$

$$g = \frac{1}{3}(126) = 42$$

$$c = 126 - 40 = 86$$

cats = 86, dogs = 126, guinea pigs = 42

15

$$U + C + J = 32.2$$

$$U = 12.2 + C$$

$$C = 3.2 + J$$

$$J = C - 3.2$$

$$12 + C + C + C - 3 \cdot 2 = 32.2$$

$$3C + 8.8 = 32.2$$

$$3C = 23.4$$

$$C = 7.8$$

$$J = 7.8 - 3.2 = 4.6$$

$$U = 12 + 7.8 = 19.8$$

U.S. = 19.8 million barrels
China = 7.8 million barrels
Japan = 4.6 million barrels

16.

$$x - y = 4$$

$$x = 4 + y$$

$$x^2 + y^2 = 25$$

$$(4 + y)^2 + y^2 = 25$$

$$16 + 8y + y^2 + y^2 = 25$$

$$2y^2 + 8y - 9 = 0$$

$$y_1, y_2 = \frac{-4 \pm \sqrt{64+72}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{136}}{4}$$

$$= \frac{-4 \pm \frac{2\sqrt{34}}{1}}{4}$$

$$= \frac{-4 \pm \sqrt{34}}{2}$$

$$\text{When } y = \frac{-4 + \sqrt{34}}{2}, x = 4 + \frac{-4 + \sqrt{34}}{2}$$

$$y = \frac{-4 - \sqrt{34}}{2}, x = 4 + \frac{-4 - \sqrt{34}}{2}$$

$$\begin{array}{r} 17 \quad 3x^2 - 4y^2 = 36 \\ + 2x^2 + 4y^2 = 29 \\ \hline 5x^2 = 65 \end{array}$$

$$x^2 = 13$$

$$x = \sqrt{13}, x = -\sqrt{13}$$

$$\text{When } x = \sqrt{13}, \quad 3(\sqrt{13})^2 - 4y^2 = 36$$

$$39 - 4y^2 = 36$$

$$4y^2 = 3$$

$$y^2 = \frac{3}{4}$$

$$y = -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$$

$$x = -\sqrt{13}, \quad 3(-\sqrt{13})^2 - 4y^2 = 36$$

$$39 - 4y^2 = 36$$

$$4y^2 = 3$$

$$y^2 = \frac{3}{4}$$

$$y = -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$$

$$x = \sqrt{13}, y = \frac{\sqrt{3}}{2}$$

$$x = \sqrt{13}, y = -\frac{\sqrt{3}}{2}$$

$$x = -\sqrt{13}, y = \frac{\sqrt{3}}{2}$$

$$x = -\sqrt{13}, y = -\frac{\sqrt{3}}{2}$$

$$18. \quad 2x^2 - y^2 = -16$$

$$x + y = 4$$

$$x = 4 - y$$

$$2(4-y)^2 - y^2 = -16$$

$$2(16 - 8y + y^2) - y^2 = -16$$

$$32 - 16y + 2y^2 - y^2 = -16$$

$$y^2 - 16y + 48 = 0$$

$$y^2 - 4y - 12y + 48 = 0$$

$$y(y-4) - 12(y-4) = 0$$

$$(y-4)(y-12) = 0$$

$$y = 4, y = 12$$

$$y = 4, \quad x = 4 - 4 = 0$$

$$y = 12, \quad x = 4 - 12 = -8$$

$$x = 0, y = 4$$

$$x = -8, y = 12$$

$$19. \quad x - 4y = 1$$

$$x^2 + 3y^2 = 3$$

$$x = 1 + 4y$$

$$(1 + 4y)^2 + 3y^2 = 3$$

$$1 + 8y + 16y^2 + 3y^2 = 3$$

$$19y^2 + 8y - 2 = 0$$

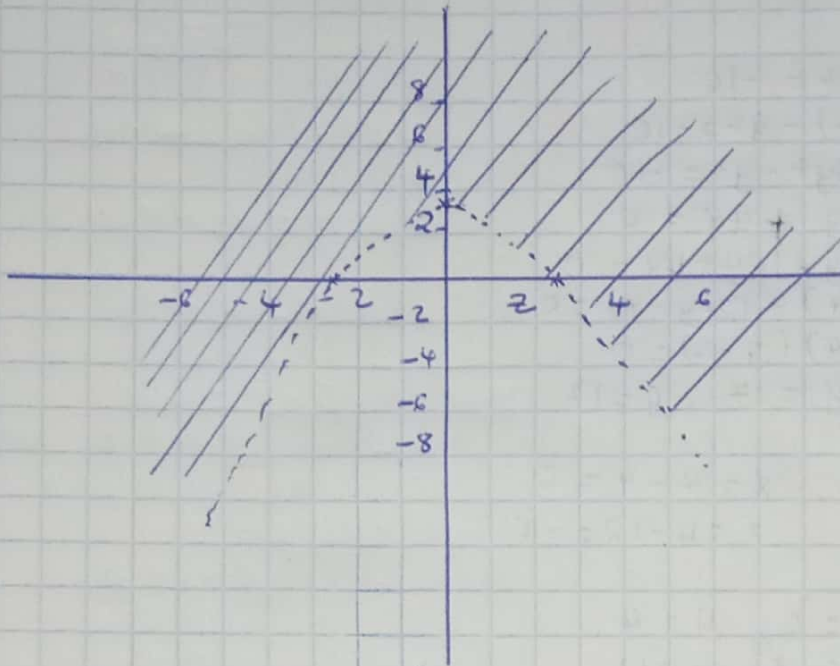
$$y_{1,2} = \frac{-8 \pm \sqrt{64 + 152}}{38} = \frac{-8 \pm \sqrt{216}}{38}$$

$$= \frac{-4 \pm 3\sqrt{6}}{19}$$

$$y = \frac{-4 + 3\sqrt{6}}{19}, \quad x = 1 + \frac{-16 + 12\sqrt{6}}{19}$$

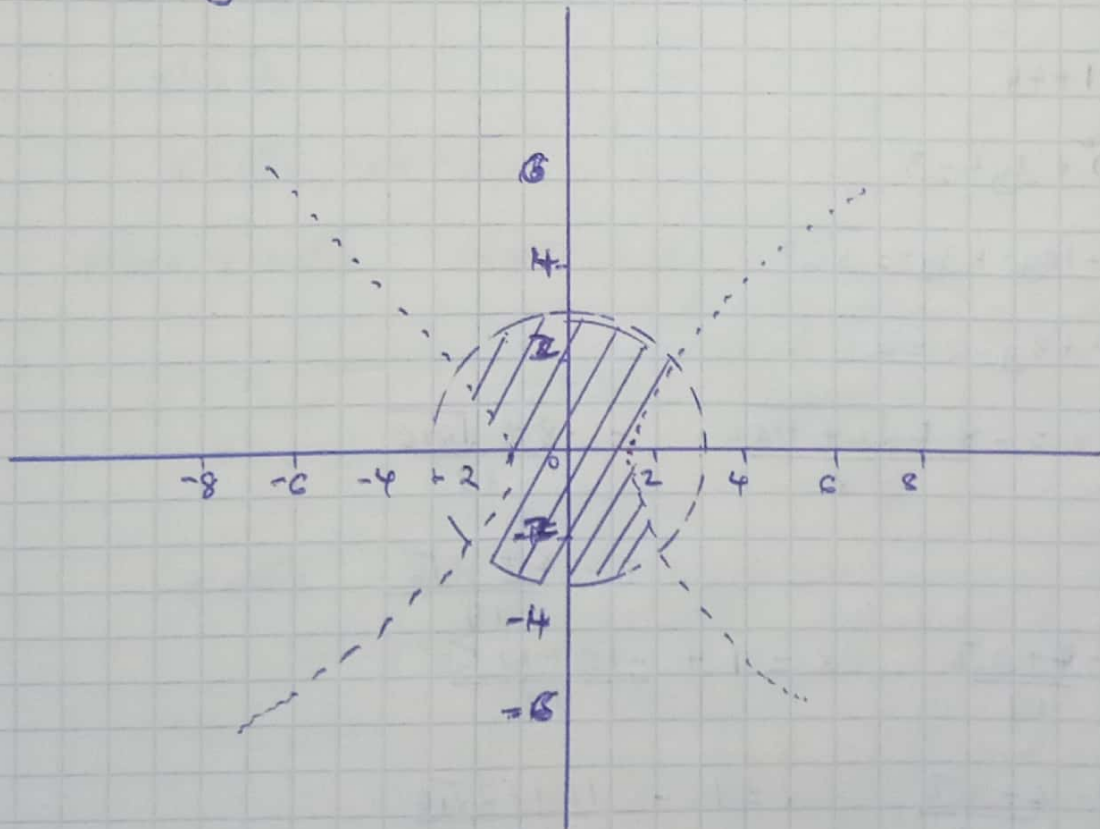
$$y = \frac{-4 - 3\sqrt{6}}{19}, \quad x = 1 - \frac{16 + 12\sqrt{6}}{19}$$

20. $x^2 + 2y > 7$

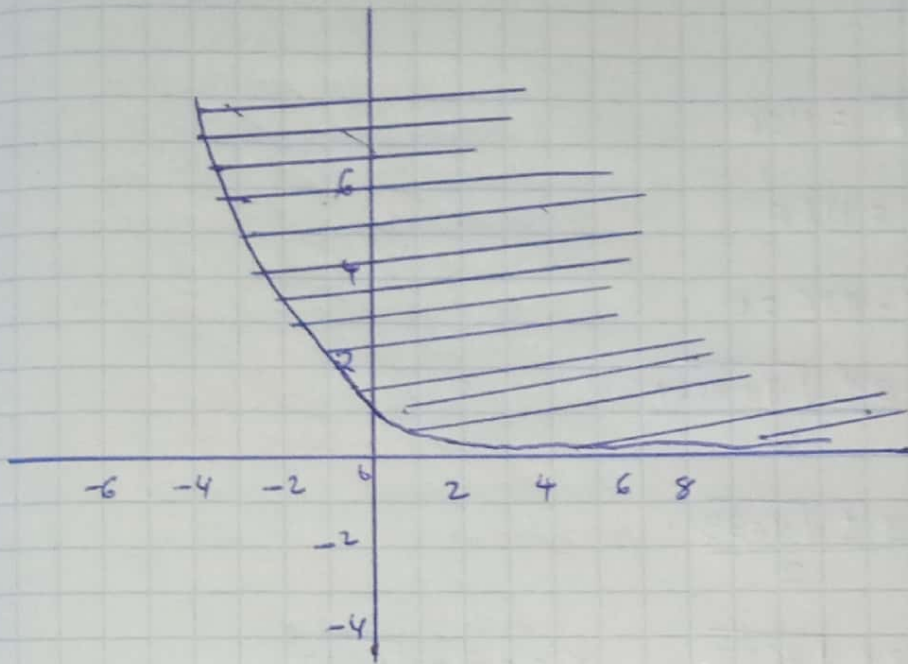


21. $y^2 - 2x^2 > -3$

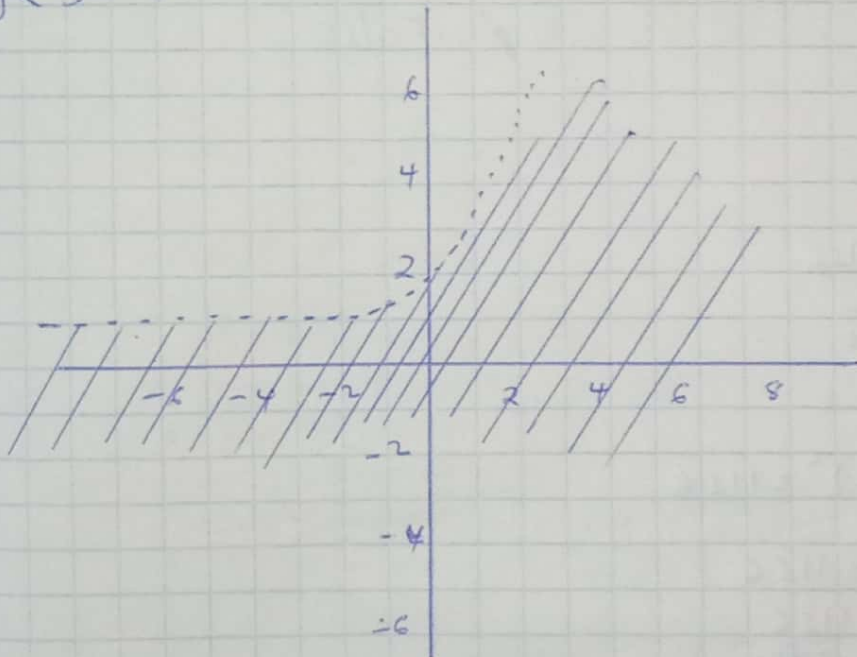
$x^2 + y^2 < 9$



22. $y > (\frac{1}{3})^x$



23. $y < 3^{2x+1}$



24. $x + y = 144$

$$y = \frac{1}{2}x^2$$

$$x + \frac{1}{2}x^2 = 144$$

$$2x + x^2 = 288$$

$$x^2 + 2x - 288$$

$$(x + 18)(x - 16) = 0$$

$$x = -18, \quad y = \frac{1}{2}(-18)^2 = 162$$

$$x = +16, \quad y = \frac{1}{2}(16)^2 = 128$$

$$25. \quad x^2 + y^2 = 1156$$

$$y = \sqrt{3}x$$

$$x^2 + (\sqrt{3}x)^2 = 1156$$

$$x^2 + 3x^2 = 1156$$

$$4x^2 = 1156$$

$$x^2 = 289$$

$$x = 17, -17$$

$$x = 17, \quad y = 17\sqrt{3}$$

$$x = -17, \quad y = 17\sqrt{3}$$